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EFFECT OF DAMPING ON THERMAL VIBRATIONS OF CIRCULAR PLATE

Narender Kumar Sarswat
Department of Physics (PG)
J. V. Jain (PG) College
Saharanpur. U. P. (India).

ABSTRACT:
The analysis presented here is to study the damping effect on thermal vibrations of an isotropic elastic circular plate of uniform thickness. The frequencies, deflections, and moments corresponding to first four modes of vibration have been computed for the two combinations of boundary conditions, clamped (C) and simply supported (SS) and various values of thermal gradient and damping parameter by applying the method of Frobenius for the solution of the governing differential equation of motion.

KEY WORDS:
Young modulus, Thermal gradient, Damping parameter, Frequency parameter, Deflections and Moments

1. INTRODUCTION:

The damping effect can be large enough to check the vibrations to produce of any appreciable effect on frequency as well as amplitude of vibrations. In recent years, interest in the effect of temperature on solid bodies has highly increased because of rapid developments in space technology. In these problems, the thermal dependence of frequency of plates of different shapes is of great importance and designing many scientific devices. The effect of temperature on the modulus of elasticity of materials is far from negligible, especially in the design of aircrafts and rockets in which certain parts have to operate under elevated temperatures. The first comprehensive collection of solutions for circular plate of uniform thickness has been presented by Airey [1] in 1911. The vibrations of circular plates of variable thickness have been studied by many authors [2, 3, 4, 5]. Tomar and Tiwari [6] have studied the effect of linearly transient temperature field on frequencies of an isotropic circular plate of linearly varying thickness. Recently, Tomar and Gupta [8, 9] considered the effect of harmonic and linearly temperature variations on axisymmetric vibrations of orthotropic circular plates of variable thickness respectively. The object of this study is to determine the effect of the parabolic temperature distribution on the frequencies of an isotropic circular plate of uniform thickness with damping effect. The differential equation of motion is solved by Frobenius method. The frequency parameters, deflections and moments corresponding to the first four modes of vibration for clamped as well as simply-supported plates have been computed for various values of damping parameters and temperature gradients with constant Poisson's ratio. The numerical results have been presented in graphical forms and compared with Jain [3]. The present investigations are helpful in designing many scientific devices where uniform structures are exposed to high intensity heat fluxes due to which the material properties undergo significant change in vibrations.

2. ANALYSIS:

The plate material is assumed to be subjected to a parabolic temperature distribution (in R-direction) is given by along the length.

$$T = T_0 (1-R^2) \qquad \text{..... (1)}$$

Where T and T_0 denotes the temperature excess above the reference temperature at any point R and at the end R = 0 respectively.
The temperature dependence of the modulus of elasticity for most of the enginering materials is given by [10, 11, 12], therefore, one can have

$$\overline{E}(T) = \overline{E}_0 (1 - \xi T) \qquad \dots (2)$$

Where $\overline{E}_0$ is the modulus of elasticity of the material at the reference temperature i.e. at T = 0 and ξ is a constant.

Taking the reference temperature, the temperature at R=1, the modulus variation becomes

$$\overline{E}(R) = \overline{E}_0 \left\{1 - \eta\left(1 - R^2\right)\right\} \qquad \dots\dots (3)$$

Where $\eta = \xi T_0, (0 \le \eta < 1)$, a parameter known as temperature gradient, and ξ is an arbitrary constants.
The governing equation of motion in non-dimensional variables is given by

$$[\overline{E}]\frac{\partial^4 W}{\partial R^4} + 2\left[\frac{\partial \overline{E}}{\partial R} + \frac{\overline{E}}{R}\right]\frac{\partial^3 W}{\partial R^3} + \left[\frac{\partial^2 \overline{E}}{\partial R^2} + \frac{(2+v)}{R}\frac{\partial \overline{E}}{\partial R} - \frac{\overline{E}}{R^2}\right]\frac{\partial^2 W}{\partial R^2}$$

$$+\left[\left(\frac{v}{R}\right).\frac{\partial^2 \overline{E}}{\partial R^2} - \frac{1}{R^2}\frac{\partial \overline{E}}{\partial R} + \frac{1}{R^3}\overline{E}\right]\frac{\partial W}{\partial R}$$

$$+12\left(1 - v^2\right)\left(\frac{a^2 \overline{\rho}}{H^2}\right)\frac{\partial^2 W}{\partial t^2} + 12\left(1 - v^2\right)\left(\frac{k}{H^3}\right)\frac{\partial W}{\partial t} = 0 \qquad \dots(4)$$

Where,

$R = \frac{r}{a}$, $W = \frac{w}{a}$, $\overline{E} = \frac{E}{a}$, $H = \frac{h}{a}$ and $\overline{\rho} = \frac{\rho}{a}$, and a is the width of the plate. With considering the mass during of the plate materials are constants.
Using equation (3), equation (4) reduces to

$$\left[1 - \eta(1 - R^2)\right]\frac{\partial^4 W}{\partial R^4} + 2\left[\{2\eta R\} + \left\{\frac{1 - \eta\left(1 - R^2\right)}{R}\right\}\right]\frac{\partial^3 W}{\partial R^3}$$

$$+\left[\{2\eta\} + (2+v)(2\eta) - \left\{\frac{1 - \eta\left(1 - R^2\right)}{R^2}\right\}\right]\frac{\partial^2 W}{\partial R^2}$$

$$+\left[\left(\frac{2\eta v}{R}\right) - \left\{\frac{2\eta}{R}\right\} + \left\{\frac{1 - \eta\left(1 - R^3\right)}{R^3}\right\}\right]\frac{\partial W}{\partial R} + \left[\frac{12\left(1 - v^2\right)a^2\overline{\rho}}{\overline{E}_0 H^2}\right]\frac{\partial^2 W}{\partial t^2}$$

$$+\left[\frac{12\left(1 - v^2\right)k}{\overline{E}_0 H^3}\right]\frac{\partial W}{\partial t} = 0 \qquad \dots\dots(5)$$

3. SOLUTION:

The harmonic damped vibration is considered

$$W(R,t)=\overline{W}(R)\ e^{-rt}\ \text{Cos}\ pt \qquad .....(6)$$

Imposing the equation (6) on equation (5) the result is.

$$\left[\left\{1-\eta+\eta R^2\right\}\right]\frac{\partial^4\overline{W}}{\partial R^4}+2\left[\left\{3\eta R\right\}+\left\{\frac{1-\eta}{R}\right\}\right]\frac{\partial^3\overline{W}}{\partial R^3}$$

$$+\left[\left\{(2v+5)\eta\right\}-\left\{\frac{1-\eta}{R^2}\right\}\right]\frac{\partial^2\overline{W}}{\partial R^2}+\left[\left\{\frac{(2v-1).\eta}{R}\right\}+\left\{\frac{1-\eta}{R^3}\right\}\right]\frac{\partial\overline{W}}{\partial R}$$

$$-\left[\left\{D_k^2\,I^{+2}+\Omega^2 I^2\right\}\right]\overline{W}=0 \qquad ....(7)$$

Where,

$$I^*=\left\{\frac{1}{H^2}\right\},\ D_k^2=\left\{\frac{3k^2\left(1-v^2\right)}{\overline{\rho}\,a^2\,\overline{E}_0}\right\}\ \&\Omega^2\left\{\frac{12\left(1-v^2\right)\overline{\rho}\,a^2p^2}{\overline{E}_0}\right\}$$

Where, p is the circular frequency , r is the frequency parameter and D_k is the damping parameter.

A series solution for $\overline{W}$ is assumed in the form

$$\overline{W}(R)=\sum_{\lambda=0}^{\infty}a_\lambda R^{c+\lambda}\quad \text{with}\ a_0\neq 0 \qquad .... (8)$$

Where C is the exponent of singularly. If the series expression (8) is substituted into the equation (7), one obtains

$$\sum_{\lambda=0}^{\infty}a_\lambda\left[b_\lambda^{(3)},\,T_1^{(1)}+b_\lambda^{(2)},\,T_1^{(2)}+b_\lambda^{(1)},\,T_1^{(3)}+b_\lambda,\,T_1^{(4)}\right]R^{C+\lambda-4}$$

$$\sum_{\lambda=0}^{\infty}a_\lambda\left[\left\{b_\lambda^{(3)},\,T_2^{(1)}+b_\lambda^{(2)},\,T_2^{(2)}+b_\lambda^{(1)},\,T_2^{(3)}+b_\lambda^{(1)},\,T_2^{(4)}\right\}\right]R^{C+\lambda-3}+\sum_{\lambda=0}^{\infty}a_\lambda\left[T_3^{(5)}\right]R^{C+\lambda}=0 \quad ...(9)$$

Where

$$T_1^{(1)}=(1-\eta),T_1^{(2)}=2(1-\eta),\ T_1^{(3)}=-(1-\eta),\ T_1^{(4)}=(1-\eta),$$

$$T_1^{(1)}=\eta,T_2^{(2)}=6\eta,\ T_2^{(3)}=\eta(2v+5),\ T_2^{(4)}=\eta(2v-1),\ T_3^{(5)}=-\left[D_k^2 I^{*2}+\Omega^2 I^*\right],$$

$$b_\lambda^3=(C+\lambda)(C+\lambda-1)(C+\lambda-2)(C+\lambda-3),$$

$$b_\lambda^{(2)}=(C+\lambda)(C+\lambda-1)(C+\lambda-2),\ b_\lambda^{(1)}=(C+\lambda)(C+\lambda-1),$$

and $b_\lambda=(C+\lambda)$

For the series expression (8) to be the solution, the coefficients of the powers of R in the equation (9) must be identically zero. Thus, by equating to zero the coefficient of the lowest power of R , the following indicial roots are obtained : C=0, 0, 2, 2. Equating to zero the coefficient of the next higher power of R, one finds that $a_1=0=a_3$ and a_2 is indeterminate for C = 0, Hence a_2 can be written as an arbitrary constant along with a_0. Similarly equating to zero

the coefficient of other higher power of R . The remaining constants $a_\lambda(\lambda=4, 5, 6, 7.........)$ are determined from the recurrence relation :

$$\left[(C+\lambda+4)^2\ (C+\lambda+2)^2\, T_1^{(1)}\right] a_{\lambda+4} + (C+\lambda+2)\left[(C+\lambda+1)\right]$$

$$\left\{(C+\lambda)(C+\lambda-1)T_2^{(1)} + (C+\lambda)T_2^{(2)} + T_2^{(3)}\right\} + T_2^{(4)}\,]\,a_{\lambda+2} + \left[T_3^5\right] a_\lambda = 0 \qquad .... (10)$$

If the notation

$$a_\lambda = U_\lambda a_0 + V_\lambda a_2 \qquad \text{with } \lambda = 0, 1, 2, 3, ... \qquad ...(11)$$

are introduced , the solution for w , corresponding to C = 0 , is

$$\overline{W} = a_0\left[1+\sum_{\lambda=3}^{\infty} U_\lambda R^2\right] + a_2\left[R^2 + \sum_{\lambda=3}^{\infty} V_\lambda R^2\right] \qquad .... (12)$$

Where, U_λ and V_λ are the functions of η, v, D_k, I^* and Ω. It is evident that no new solution will arise corresponding to other values of C i.e. for C=2, as it is already contained in the solution (12) with the arbitrary constants a_0 and a_2. Using the technique used by Lamb [7] to test the convergence, one finds that the solution (12) is uniformly convergent in the interval $0 \le R \le 1$, where $|\mu| < 1$. Hence the solution is convergent for all $\eta < 0.5$, where $\mu = \lim\text{it}_{\lambda\to\infty}\left(\frac{a_{\lambda+1}}{a_\lambda}\right)$.

4. BOUNDARY CONDITIONS AND FREQUENCY EQUATIONS:

The frequency equations for clamped and simply supported circular plates have been obtained by employing the appropriate boundary conditions :

CLAMPED PLATE:

For a circular plate clamped at the edge r = a, the deflection W and the slope of the plate element at the edge should be zero.

$$\text{i.e. } W(r,t)\Big|_{r=a} = \frac{\partial W(r,t)}{\partial R}\Big|_{r=a} = 0 \text{ or } \overline{W}\Big|_{R=1} = \frac{d\,\overline{W}}{d\,R}\Big|_{R=1} = 0 \qquad ...(13)$$

Using equation (12) and boundary conditions (13) one gets

$$a_0\left[1+\sum_{\lambda=3}^{\infty} U_\lambda\right] + a_2\left[1+\sum_{\lambda=3}^{\infty} V_\lambda\right] = 0 \text{ and } a_0\left[\sum_{\lambda=3}^{\infty}\lambda U_\lambda\right] + a_2\left[2+\sum_{\lambda=3}^{\infty}\lambda V_\lambda\right] = 0$$

Eliminating the unknown constants a_0 and a_2, one obtains the frequency equation for clamped plate as :

$$\begin{vmatrix} Q_1(\Omega) & Q_2(\Omega) \\ Q_3(\Omega) & Q_4(\Omega) \end{vmatrix} = 0 \qquad \text{..... (14)}$$

Where

$$Q_1(\Omega) = \left[1 + \sum_{\lambda=3}^{\infty} U_\lambda\right], \qquad Q_2(\Omega) = \left[1 + \sum_{\lambda=3}^{\infty} V_\lambda\right],$$

$$Q_3(\Omega) = \left[x + \sum_{\lambda=3}^{\infty} \lambda U_\lambda\right], \qquad Q_4(\Omega) = \left[2 + \sum_{\lambda=3}^{\infty} \lambda V_\lambda\right],$$

SIMPLY SUPPORTED PLATES:

For a circular plate simply supported at the edge r=a, the deflection W and the moment M_r at the edge should be zero

i.e. $$W(r,\ t)\big|_{r=a} = M_r(r,\ t)\big|_{r=a} = 0$$

or $$\overline{W}\Big|_{R=1} = \left[\frac{d^2\overline{W}}{dR^2} + \frac{v}{R}\frac{d\overline{W}}{dR}\right]\Bigg|_{R=1} = 0 \qquad \text{..... (15)}$$

Again using equation (12)and boundary conditions (15) one gets : i.e.

$$a_0\left[1 + \sum_{\lambda=3}^{\infty} U_\lambda\right] + a_2\left[1 + \sum_{\lambda=3}^{\infty} V_\lambda\right] = 0$$

And

$$a_0\left[\sum_{\lambda=3}^{\infty} \lambda(\lambda + v - 1)U_\lambda\right] + a_2\left[2(1+v) + \sum_{\lambda=3}^{\infty} \lambda(\lambda + v - 1)V_\lambda\right] = 0$$

Eliminating the unknown constants a_0 and a_2, one obtains the frequency equation for simply supported plate as:

$$\begin{vmatrix} Q_1(\Omega) & Q_2(\Omega) \\ Q_5(\Omega) & Q_6(\Omega) \end{vmatrix} = 0 \qquad \text{..... (16)}$$

Where

$$Q_5(\Omega) = \left[\sum_{\lambda=3}^{\infty} \lambda(\lambda + v - 1)U_\lambda\right]$$

and

$$Q_6(\Omega) = \left[2(1+v) + \sum_{\lambda=3}^{\infty} \lambda(\lambda + v - 1)V_\lambda\right]$$

5. DEFLECTION FUNCTION AND MOMENTS:

Using the boundary condition $\overline{W}=0$ at R = 1 and taking a_0 = 1, in equation (12) one obtains :

$$\overline{W}=\left(1+\sum_{\lambda=3}^{\infty}U_\lambda R^\lambda\right)-\left[\frac{\left(1+\sum_{\lambda=3}^{\infty}U_\lambda\right)}{1+\sum_{\lambda=3}^{\infty}V_\lambda}\right]\left(R^2+\sum_{\lambda=3}^{\infty}V_\lambda R^\lambda\right) \quad . \quad .... (17)$$

Also , $$M_r=-D\left[\frac{\partial^2 w}{\partial r^2}+\frac{v}{r}\frac{\partial w}{\partial r}\right] \quad .....(18)$$

Again using the boundary condition $\overline{W}$ = 0 at R =1 and taking a_0 = 1, we obtained the non-dimensional moment parameter equation.

6. RESULTS AND DISCUSSION:

Numerical results for an isotropic, elastic circular plate have been computed from the equations (14), (16), (17) and (18) when the temperature field varies as parabolically. In all the cases , the Poisson's ratio has been taken to be 0.3 and terms of the series upto an accuracy of 10^{-8} in their absolute values have been retained. If the η = 0= D_K are considered , the results so obtained are in good agreement with the results obtained by Jain [3]. The results for an isotropic elastic circular plate of uniform thickness have been computed for four modes. However the results of first two modes have been shown in the figures. It is observed (fig. 1and 2) that the frequency Ω decreases with the increase of damping parameter D_K for heated as well as unheated (η = 0) plates. For higher values of D_K the fall in frequency parameters Ω is very sharp repidly specialy for simply supported edge conditions when the values of η is higher(for 3&4 mode). Furthermore, the frequencies of heated plates are lower than that of unheated(η = 0) one in both the cases of boundary conditions . Observation form fig. 3 and 4 the values of Ω decreases with the increase of thermal gradient η in both the cases of boundary conditions. It is also noted that the transverse displacements $\overline{W}$ are less for simply supported plate than that for clamped plate for all the four modes of vibrations. The frequency parameters Ω for clamped-plates is higher than the corresponding to simply supported -plates for all the four modes. The deflection function $\overline{W}$ and moment parameters $\overline{M}$ have been computed for heated and damped plates with clamped and simply supported edge conditions corresponding to the first four modes of vibration have been plotted in figures 5 and 6 respectively. It's clear from graph the $\overline{W}$ are less for simply supported plates than that for clamped plate for all the four modes of vibrations.

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Fig. 1: Variation Of Frequency Parameter 'Ω' 'With Damping Parameter ' D_k' For A Circular Plate Corresponding To First Mode Of Vibration Under The Parabolic Thermal Gradient ' H ' .

LEGEND: H=0.1, ν = 0.3, C = Clamped , SS = Simply Supported

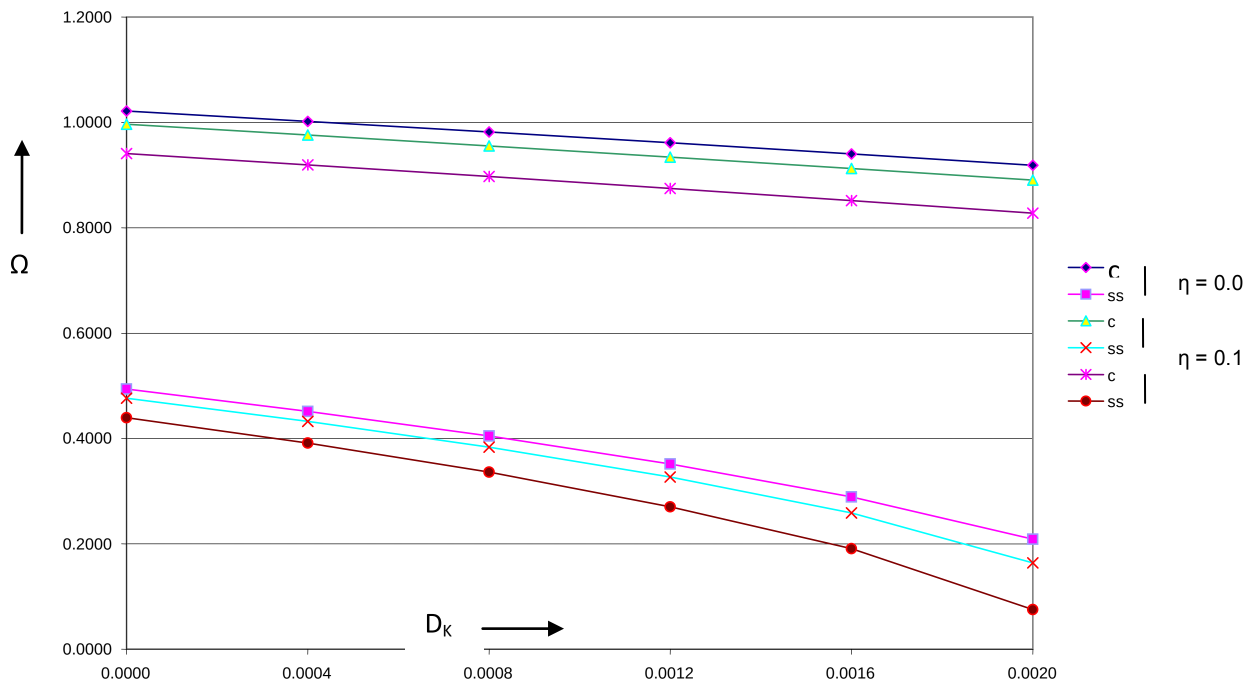

Fig. 2: Variation Of Frequency Parameter 'Ω' With Damping Parameter ' D_k' For A Circular Plate Corresponding To Second Mode Of Vibration Under The Parabolic Thermal Gradient ' H '.

LEGEND: H=0.1, ν = 0 .3, C = Clamped , SS = Simply Supported

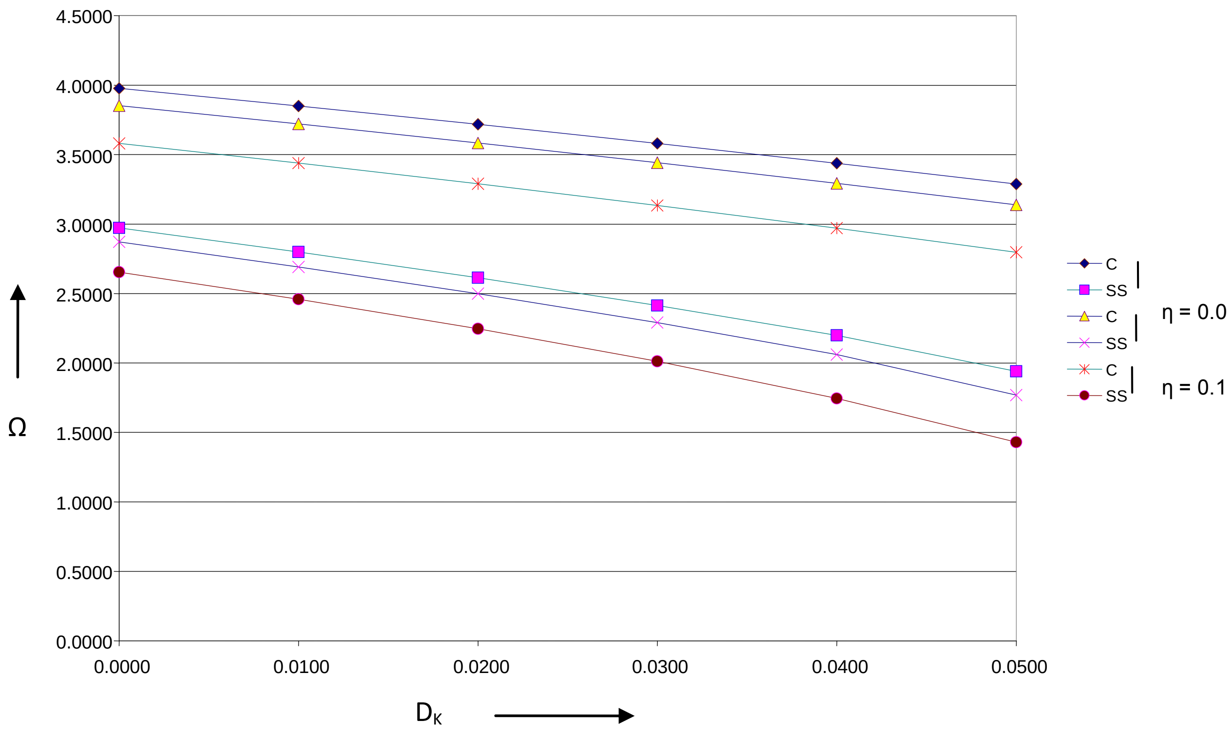

Fig. 3: Effect Of Parabolic Thermal Gradient ' H ' On The Frequency Parameter ''Ω' Of A Circular Damped Plate Corresponding To First Mode Of Vibrations.

LEGEND: =0.1, ν = 0.3, C = Clamped , SS = Simply Supported

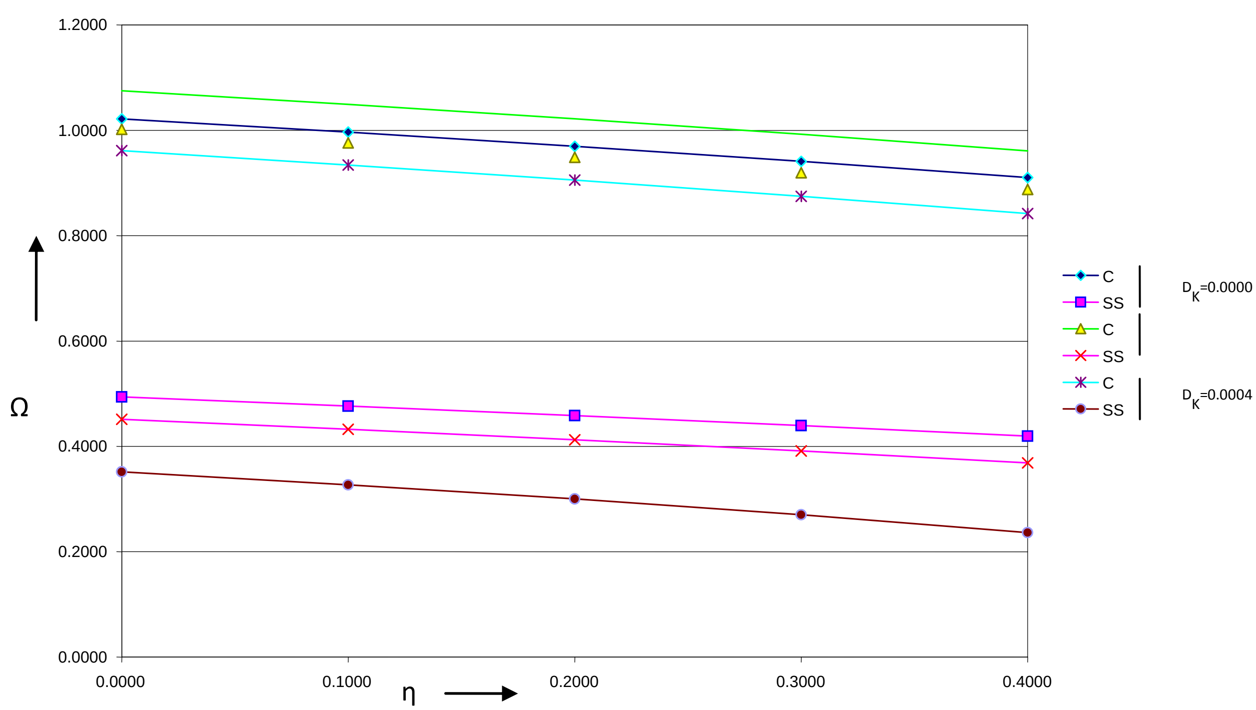

Fig. 4: Effect Of Parabolic Thermal Gradient' H ' On The Frequency Parameter 'Ω' Of A Circular Damped Plate Corresponding To Second Mode Of Vibrations.

LEGEND: H=0.1, ν = 0.3, C = Clamped , SS = Simply Supported

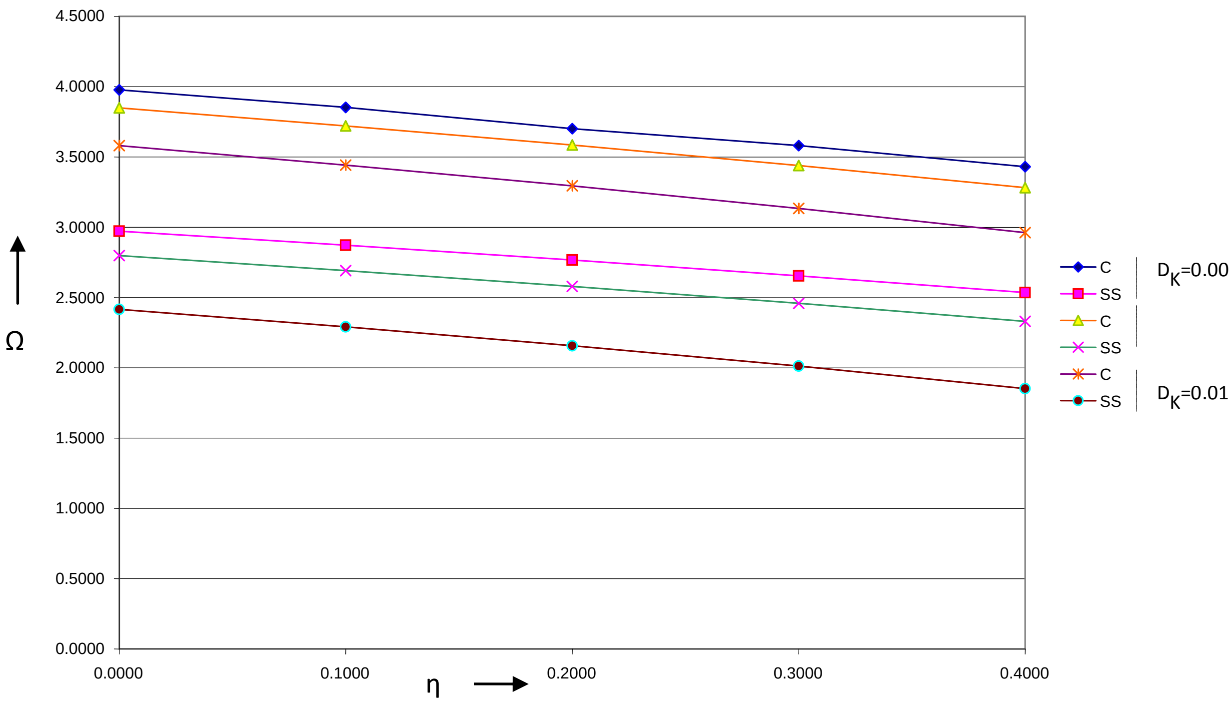

Fig. 5: Transverse Displacement 'Ŵ' At Different Point's Of An Circular Damped Plate For The First Four Modes Of Vibrations Under The Parabolically Temperature Field

LEGEND: H=0.1,ν = 0.3, η = 0.1 C = Clamped , SS = Simply Supported

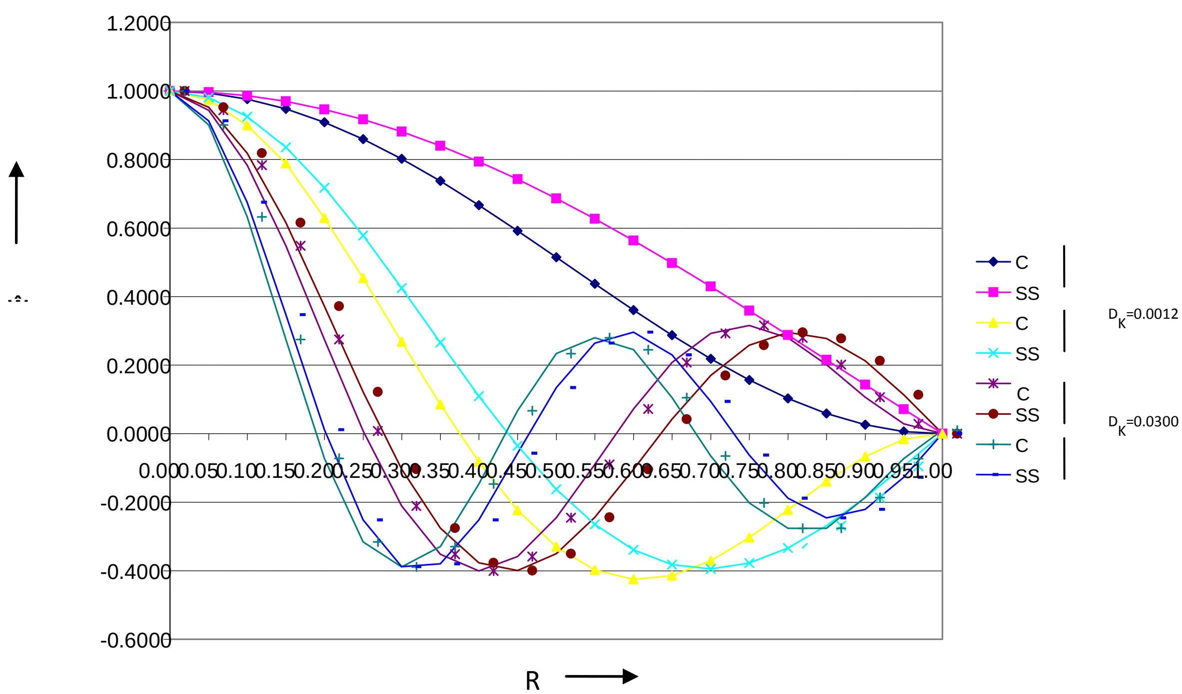

Fig. 6: Moment Parameters 'M' At Diferrent Point's Of An Circular Damped Plate For The First Four Modes Of Vibrations Under The Parabolically Temperature Field

LEGEND: H=0.1,ν = 0.3, η = 0.1 C = Clamped , SS = Simply Supported